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INFLATION FORECASTS USING THE TIPS YIELD CURVE

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Abstract

In this paper month-on-month inflation is forecasted using information on expected inflation by participants in financial markets as an additional regressor to a direct autoregressive method and the results are compared to those of the most commonly used univariate forecasting methods. These forecasts are then used to forecast quarterly inflation and the results are compared to those of the SPF and iterated autoregressive with fixed number of lags.

Keywords: Inflation Forecasting, Treasury Inflation Protected Securities (TIPS), Survey of Professional Forecasters

1. Introduction

Treasury Inflation Protected Securities (TIPS) are debt instruments issued by the U.S. Treasury with principal payments indexed to the seasonally unadjusted Consumer Price Index for Urban Consumers (CPI-U). They were firstly issued in 1997 with two main purposes: to fully protect investors from inflation (and consequent losses in purchasing power) and to provide daily data on future inflation market expectations to the Federal Reserve. TIPS are the only “riskless” financial instruments offering full protection against inflation to investors. Similarly to nominal U.S. securities, TIPS are traded in secondary markets, making it possible to infer daily expectations of future average inflation from market data.

During the initial years, the TIPS market was characterized by poor liquidity conditions and consequently low trading volumes, mainly due to the fact that investors were not familiar with TIPS and had difficulties in evaluating them. As investors became more familiar with the characteristics of TIPS and the Federal Reserve announced its commitment to the TIPS program in 2003, the liquidity of the TIPS market improved and the trading volume of TIPS increased significantly. Currently the TIPS market average daily trading volume is 11,5 billion U.S. dollars and in 2008 it accounted for 11% of the total U.S. government debt. They are traded mostly by investors with long-term goals like pension funds, some hedge funds and insurance companies.

For a given maturity, the spread between yields of nominal and inflation-indexed bonds (real yields) is called inflation compensation as it measures the compensation required by investors of nominal securities, due to the inflation risk, above the real component of nominal yields. This spread is also known as the breakeven inflation (BEI) since it represents the level of inflation that, if realized, will provide the same return to investors in nominal and inflation-indexed securities and would make investors indifferent between these different Treasury securities. From Fisher's relation we can interpret the BEI as the market expectation of the average annual inflation between today and the maturity of the securities.

In this paper we forecast U.S. inflation incorporating these expectations of future inflation contained in the yield spread into augmented autoregressive models and evaluate how they perform compared to other methods. "Since investors suffer financial losses when their forecasts err, it seems reasonable to assume that market participants will try to forecast future inflation as accurately as possible" (Shen and Corning (2001)) and, taking into account that inflation expectations affect people's behavior and drive prices, these expectations could contain useful information to forecast inflation.

This Work Project is organized as follows. In section 2 we present a literature review on inflation forecasting and the motivation for this Work Project. Section 3 describes TIPS, their market and breakeven inflation. Section 4 presents the forecasting methods used. Section 5 summarizes the results and section 6 concludes and provides suggestions for further research.

2 Literature Review

According to Stock and Watson (2007) in the last thirty years "inflation in the United States has become both easier and harder to forecast" as inflation has become much more stable since then, making it significantly easier to forecast, but, on the other hand, it has become increasingly difficult to outperform simple univariate forecasting methods. Ang et al. (2007) have shown that survey forecasts, such as the Survey of Professional Forecasters (SPF) and the Livingston survey outperform the best univariate forecasts and conjecture that it is due to three reasons: "the pooling of large amounts of information; the efficient aggregation of that information; and the ability to quickly adapt to major changes in the economic environment."

To the best of our knowledge, the implicit inflation expectations extracted from the TIPS yield curve have only been used directly as forecasts and compared to survey expectations average inflation as in Shen and Corning (2001). Quoting Stock and Watson (2008), "We are not aware of any papers that evaluate the performance of inflation forecasts backed out of the TIPS yield curve, and such a study would be of considerable interest." We fill this gap by forecasting inflation using expectations of inflation extracted from the TIPS yield curve as additional regressors in augmented autoregressive models.

3 Inflation-indexed Securities

3.1 TIPS

TIPS are bonds whose principal is indexed to the level of CPI-U and the interest rate of semi-annual coupon payments is fixed. In other words, when the price level increases investors receive higher payments (in case of deflation, investors are protected with a floor that guarantees them a payment of at least the original principal value). Being issued by the U.S. Treasury, TIPS are the only riskless securities capable of providing investors a full hedge against high inflation by indexing the return to the price level.[1]

TIPS are issued with long maturities (initially TIPS were issued with 5, 10, and 30 years but between 2004 and 2009 20-year TIPS were issued) in regular auctions

[1] TIPS are also denominated real bonds as they provide returns in terms of the amount of goods and services that investors can purchase.

throughout the year and are traded daily on secondary markets[2]. Compared to the U.S. government nominal bonds, TIPS offer the advantage of full protection against inflation to investors. On the other hand, TIPS have the disadvantage of being less liquid debt securities than nominal bonds with similar maturities and investors require a compensation for this.

From the point of view of the Federal Reserve and U.S. policy makers, TIPS provide various potential benefits. The data on daily trading provides regular information on inflation expectations by a large number of market agents which is essential to conduct successful monetary and fiscal policies. The issuance of TIPS may also "give the Treasury access to a broader investor base and reduce the Treasury's overall borrowing costs" (Dudley et al. (2009)).

3.2 The TIPS Market

When inflation-indexed securities were firstly issued in 1997 by the U.S. Treasury the TIPS market was characterized mainly by poor liquidity conditions. As investors became more familiar with these securities and the U.S. Treasury announced full commitment to the TIPS program in 2002 (Dudley et al. (2009)) the trading volume increased and liquidity conditions improved significantly since 2003 (D'Amico et al. (2010)). [3] However, in 2012 the TIPS market remains significantly less liquid than the U.S. nominal debt market. Shen (2009) attributes this persistent lower liquidity of the TIPS market to three reasons: investors do not fully understand the mechanisms of TIPS and find it hard to valuate these instruments properly; most trading of government nominal debt securities is done for hedging of risky portfolios, as they provide fixed

[2] 5-year TIPS are auctioned in April, August, and December; 10-year TIPS are auctioned with more frequency in January, March, May, July, September, and November; and 30-year TIPS in February, June, and October.

[3] The commitment to the TIPS program was reaffirmed in August 2008 (Dudley et al. (2009)).

periodical returns in contrast to TIPS variable inflation-indexed payments that make them inappropriate for hedging purposes; and most investors of the TIPS markets have long-term goals and generally hold their TIPS until maturity instead of selling them. [4]

Improving liquidity of inflation-indexed debt markets is "necessary to fully capture the benefits of inflation-indexed Government securities. Without such markets, there would be a sizable liquidity premium in the yield of the securities" (Shen (2009)). One of the most liquid indexed inflation debt markets is the UK's Gilts market that currently accounts for more than 30% of total UK government debt. Having started in 1981, it is a much more mature market than the U.S. TIPS market and according to the appendix "What Can We Learn from the UK Experience?" of Shen (2001) there is strong evidence of considerably higher liquidity in the UK inflation-indexed securities market compared to the TIPS market and "the experience of the UK suggests that if the U.S. Treasury keeps issuing inflation- indexed Treasuries and their liquidity continues to improve, the liquidity premium will decline over time." (Shen (2001)).

3.3 The TIPS Yield Curve and Breakeven Inflation

Using a similar methodology to the one used in their previous paper "The U.S. Treasury Yield Curve: 1961 to the Present", Gurkaynak et al. (2008) computed a smoothed TIPS yield curve with daily data from outstanding off-the-run TIPS and made the data available at the Research Data section of the Board of Governors of the Federal Reserve System's webpage.[5]

For a given maturity, the spread between the yields of nominal securities and TIPS can be interpreted as an inflation expectation. This derives directly from Fisher's hypothesis rearranged into an ex ante expectations augmented equation:

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[4] Long-term investors include insurance companies, pension and endowment funds.
[5] http://www.federalreserve.gov/econresdata/researchdata.htm

$$\pi^e = y^n - y^r.$$

This relation can be represented graphically:

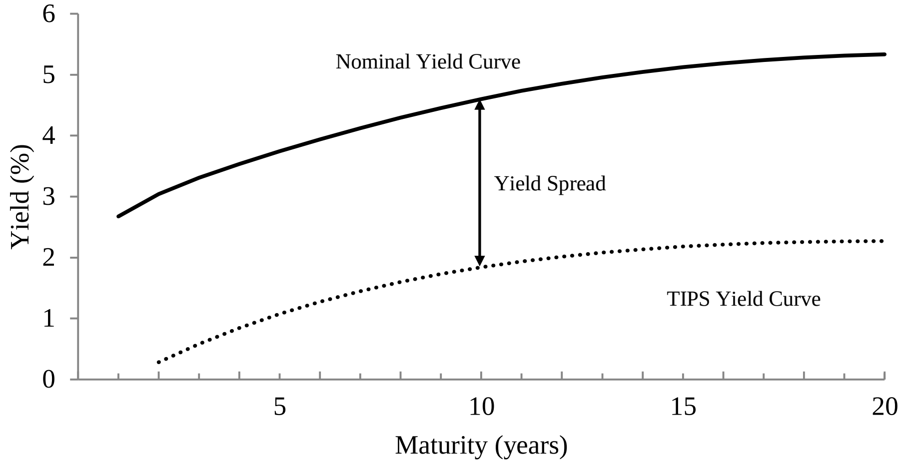

Figure 1: Nominal and TIPS yield curve of 2/12/2004

For the date of 2/12/2004, the yield spread between nominal and indexed bonds maturing in 10 years is $4{,}60 - 1{,}84 = 2{,}76$. This can be interpreted as follows: for a maturity of 10 years a nominal bond offers investors a yield of $4{,}60$, a TIPS offers investors a yield of $1{,}84$ and the expected average annual inflation between 2/12/2004 and the 2/12/2014 is $2{,}76$ percentage points . From the definition of breakeven inflation (BEI) stated in section 1, we can also think of $2{,}76$ as the level of average annual inflation that, if realized, would offer the same return to investors in TIPS and investors in nominal bonds.

The Fisher relation, despite being a good approximation, does not hold necessarily due to the lower liquidity of TIPS and the inflation risks incurred by investors of nominal securities. To compensate them for these risks, investors require a premium in the yields: investors in nominal securities require an inflation premium ($p(\pi)$) to compensate them for the possibility of realized inflation being high enough to significantly decrease their real gains; TIPS investors require a liquidity premium ($p(l)$) to compensate them for the lower liquidity of their securities in secondary markets. Taking these premia into account, the yield spread can be decomposed into the expected inflation π^e, the inflation premium $p(\pi)$ and the liquidity premium $p(l)$:

$$y^n - y^r = \pi^e + p(\pi) - p(l).$$

According to Gurkaynak et al. (2008) "this estimated liquidity premium is high, but it is also very inertial" and it "remained big in the early years and then gradually faded away in 2003." Estimates by Pericoli (2012) suggest a small risk premium in the initial years of the TIPS market and a considerable and variable inflation risk premium from 2001 onwards. Consequently, despite expected inflation being the main component of the yield spread, the liquidity and inflation risk premia are significant and breakeven inflation rates should not be directly interpreted as the market's expectation of average inflation.

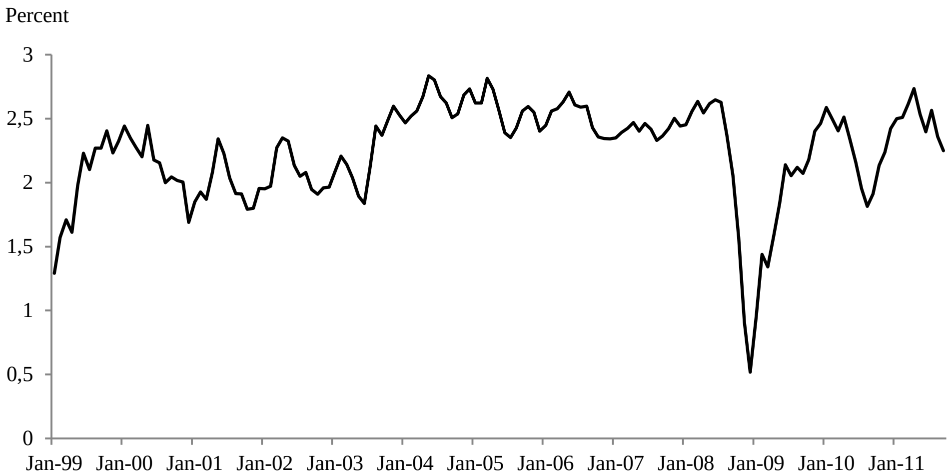

Figure 2: Breakeven inflation for securities maturing in 10 years

4 Methodology

In this work project we constructed pseudo out-of-sample forecasts of month-on-month inflation based on the TIPS yield curve by using breakeven inflation series with different maturities as additional regressors in simple autoregressive models. Comparing these forecasts to those of simple autoregressive processes we can assess the value added of the market expectations of inflation in forecasts of inflation. We forecasted the evolution of CPI at various horizons up to 2 years and computed quarterly inflation forecasts comparing them to SPF median forecasts.

4.1 Data

As stated in section 3.2, the data of the TIPS yield curve was computed by Refet S. Gurkaynak, Brian Sack, and Jonathan H. Wright with the methodology explained in Gurkaynak et al. (2008) and downloaded from the Research Data section of the Board of Governors of the Federal Reserve System's webpage.[6] This data set includes real yields and breakeven inflation rates expressed in various types of debt securities (zero-coupon, par, instantaneous forward, one-year forward, five-to-ten-year forward) and collected daily between January of 1999 and September of 2011. We opted to use zero-coupon breakeven inflation series with maturities of 5, 6,..., 19 and 20 years and converted the daily data to month-on-month data using simple averages. [7]

Realized inflation was computed using logarithmic changes in non-seasonally adjusted consumer price index for all urban consumers (CPI-U) downloaded from the Bureau of Labor Statistics of the U.S. Department of Labor webpage[8].

[6] The data set is updated on a weekly basis.
[7] There are breakeven inflation series with maturities of 2, 3 and 4 years but they are only available from 2004 onwards.
[8] http://www.bls.gov/

The forecasts of SPF were downloaded from the Federal Reserve of Philadelphia webpage[9].

4.2 Forecasts

4.2.1 Month-on-month inflation forecasts

Month-on-month inflation was forecasted using the following methods:

1. Direct autoregression (DAR) with forecast horizon *h* and *p* lags:

$$\pi_{t+h} = \beta_0 + \sum_{j=1}^{p} \beta_j \pi_{t+1-j} + \varepsilon_{t+h}$$

where π_{t+h} is inflation at $t + h$ and ε_{t+h} is the regression error. The number of lags can be fixed (FL) or determined by the Akaike Information Criterion (AIC).

2. Iterated autoregression (IAR) forecasts with forecasting horizon *h* and *p* lags:

$$\pi_{t+1} = \beta_0 + \sum_{j=1}^{p} \beta_j \pi_{t+1-j} + \varepsilon_{t+1}$$

and

$$\hat{\pi}_{t+h|t} = \hat{\beta}_0 + \sum_{j=1}^{p} \hat{\beta}_j \hat{\pi}_{t+1-j} + \varepsilon_{t+h} \text{ where } \hat{\pi}_{j|t} = \pi_j \text{ for } j \leq t$$

$\hat{\pi}_{t+h|t}$ is the forecast of inflation at $t + h$ computed using past inflation for $j \leq t$ and forecasts for $j > t$. The lag length can be fixed (FL), determined by the Akaike Information Criterion (AIC) or determined by the Bayes Information Criterion (BIC).

3. Direct augmented auto regression (DAAR) with forecasting horizon *h* and *p* lags:

$$\pi_{t+h} = \beta_0 + \sum_{j=1}^{p} \beta_j \pi_{t+1-j} + \sum_{i=1}^{r} \sum_{j=1}^{m_r} \rho_j \, BEI_{i,t+1-j} + \varepsilon_{t+h}$$

The lag length is determined by the Akaike Information Criterion (AIC).

[9] http://www.phil.frb.org/research-and-data/real-time-center/survey-of-professional-forecasters/

4. No-change forecast:

$$\pi_{t+h} = \pi_t$$

for any horizon h.

Forecasts computed with these methods were performed with $h = 1,2,4,6,8,12,24$ periods, fixed number of lags $p_{fix} = 1,2,4,6,8,10,12$ or maximum number of lags $p_{max} = 2,4,6,8,10,12$. In order to avoid erroneous results due to lack of sufficient observations to produce forecasts with long horizons three evaluation periods (defined the starting observation (SO) parameter) were used for this pseudo out-of-sample forecasts: February of 2002 until September of 2011($SO = 36$); February of 2003 to September of 2011 ($SO = 48$); February of 2004 to September of 2011 ($SO = 60$, only for forecasts with $h = 24$).

4.2.2 Quarterly inflation forecasts and comparison with SPF

The SPF is performed every quarter by a small group of professional forecasters who answer a survey elaborated by the Federal Reserve Bank of Philadelphia during the middle month of each quarter. Their individual answers are collected and statistically treated to obtain mean and median forecasts of various macroeconomic variables for the current and following five quarters. One of the forecasted variables is quarterly inflation, defined as percentage changes instead of logarithmic differences (Ang et al. (2007)) of the average quarterly levels of the seasonally adjusted CPI-U.

To produce quarterly inflation forecasts comparable to SPF forecasts we first forecasted month-on-month inflation with two methods: the Iterated Autoregression with a fixed number of lags (IAR (FL)) and the DAAR method with the expected inflation included in the TIPS yield curve we as additional regressors (we used all the breakeven inflation series with maturities ranging from 5 to 20 years). Month-on-month inflation was forecasted with $h = 1,2,\ldots,10,11$ and the maximum number of lags of 6. We then used these results to forecast the evolution of the seasonally adjusted CPI-U over the following 11 months after the initial month of each quarter. For example, in the first quarter of 2002 (2002:Q1), information of past inflation and of expected future inflation extracted from the TIPS yield curve available until January of 2002 was used to forecast February's inflation ($\hat{\pi}_{h=1}$), March's inflation ($\hat{\pi}_{h=2}$) and the inflation for the following months until December ($\hat{\pi}_{h=11}$). In order to make a fair comparison between our methodology and the SPF we took into account the fact that when professional forecasters answer the survey during the middle month of the quarter the CPI of the first month of that quarter is already known. Therefore, we used the actual value of the CPI of the first month of each quarter (CPI_t) and the forecast of inflation for the second month of the quarter ($\hat{\pi}_{h=1}$) to forecast the CPI of the second month of that quarter:

$$\widehat{CPI}_{t+1} = (1 + \hat{\pi}_{h=1})CPI_t$$

The forecast of $\widehat{CPI}_{t+1}$was used to forecast the price level of the third month of that quarter:

$$\widehat{CPI}_{t+2} = (1 + \hat{\pi}_{h=2})\widehat{CPI}_{t+1}$$

This recursive procedure was iterated to forecast the CPI level for the following months ($\widehat{CPI}_{t+3}$, $\widehat{CPI}_{t+4}$, …, $\widehat{CPI}_{t+11}$). The actual CPI and the forecasted levels of CPI were converted into quarterly (as 3-month averages) data. This procedure was performed for the period between 2002Q1 and 2011Q2. Our forecasts directly comparable to the median SPF forecasts are the percentage changes of forecasted quarterly CPI.

5 Results

5.1 Month-on-month inflation

Forecasts of month-on-month inflation (measured as logarithmic changes of the non-seasonally adjusted CPI-U) produced using the methods presented in section 4.2.1 were evaluated using the root mean square forecasting error (RMSFE).[10] For each of the simple autoregressive methods used (DAR (FL), DAR (AIC), IAR (FL), IAR (AIC) and IAR (BIC)) we constructed a benchmark by choosing the forecasts for each horizon and starting observation with the lag length corresponding to the smallest RMSFE. Forecasts from DAAR (AIC) using breakeven inflation series as additional regressors were compared to these benchmarks to infer the added value of incorporating expectations of future inflation as additional regressors to simple autoregressive forecasting methods.

Starting Observation	Horizon	DAR (FL)	DAR (AIC)	IAR (FL)	IAR (AIC)	IAR(BIC)	No-change
36	$h = 1$	0,38	0,38	0,38	0,38	0,38	0,44
	$h = 2$	0,47	0,48	0,41	0,47	0,47	0,44
	$h = 4$	0,48	0,50	0,43	0,45	0,45	0,44
	$h = 6$	0,45	0,45	0,43	0,43	0,43	0,44
	$h = 8$	0,50	0,50	0,43	0,44	0,44	0,44
	$h = 12$	0,39	0,39	0,42	0,43	0,44	0,44
	$h = 24$	0,39	0,47	0,44	0,44	0,44	0,44
48	$h = 1$	0,39	0,39	0,39	0,39	0,39	0,51
	$h = 2$	0,49	0,50	0,43	0,49	0,49	0,51
	$h = 4$	0,50	0,52	0,45	0,47	0,47	0,51
	$h = 6$	0,47	0,47	0,45	0,45	0,45	0,51
	$h = 8$	0,52	0,52	0,45	0,46	0,46	0,51
	$h = 12$	0,40	0,41	0,41	0,44	0,46	0,51
	$h = 24$	0,40	0,48	0,43	0,45	0,46	0,51
60	$h = 24$	0,41	0,48	0,45	0,47	0,47	0,54

Table 1 - RMSFE of month-on-month inflation benchmarks of autoregressive methods and the No-change forecast for the evaluation period of February of 2002 and September of 2011

[10] $RMSFE = \frac{1}{n}\sum_{i=1}^{n}(\hat{\pi}_{t+h,i} - \pi_{t+h,i})^2$ where $\hat{\pi}_{t+h,i}$ is the forecast of $\pi_{t+h,i}$.

From table 1 we can conclude that the IAR (FL) is the best benchmark forecasting method used in this exercise. Therefore, until the end of this section we will treat the IAR (FL) method as the global benchmark and present the forecasting errors as relative RMSFE, defined as, for a given horizon, the ratio of the RMSFE of the forecasting method and the RMSFE of IAR (FL).

In table 2 we present the relative RMSFE of month-on-month inflation forecasts of the simple autoregressive methods and the augmented autoregressive forecasts using the breakeven inflation series with maturity of 10 years as an additional regressor. [11]For each horizon within each evaluation period (defined by the SO) the lowest relative RMSFE is highlighted in bold. From table 2 we conclude that, despite not having lower RMSFE than the benchmark IAR (FL) for any of the horizons and evaluation periods used, the DAAR forecasts improve simple univariate forecasts (AR Benchmark) for horizons of 2, 4 and 8 periods and have very similar RMSFE than the benchmark method IAR (FL) for forecast horizons of 2, 4, 6 and 8 periods, in the two main evaluation periods (SO=36 and SO=48), unlike the No-change method in the first evaluation period (SO=36). Taking into account these consistently similar results to those of the benchmark method IAR (FL) for these horizons and the fact that these benchmarks of simple autoregressive methods were artificially created choosing for each horizon the forecasts with the lowest RMSFE of various forecasts that differ in the number of lags, we decided that the DAAR method using inflation expectations extracted from the TIPS yield curve can provide worthy forecasts of month-on-month inflation to forecast quarterly inflation with the method explained in section 4.2.2.

[11] Since all the breakeven inflation series produced very similar forecasts of year on year and month on month inflation regardless of their maturities we opted to present the results of forecasts solely due to the fact that TIPS with maturities of 10 years are the most common indexed-inflation securities.

Starting Observation (SO)	Horizon	**IAR (FL)**	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR (BIC)	No-Change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	**1,00**	1,00	1,00	1,00	1,01	1,16	1,00	1,19	1,19	1,19	1,19	1,19	1,19
	$h = 2$	**1,00**	1,16	1,16	1,14	1,14	1,07	1,16	1,09	1,13	1,15	1,13	1,13	1,15
	$h = 4$	**1,00**	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,03	1,02
	$h = 6$	**1,00**	1,05	1,05	1,00	1,01	1,02	1,05	1,03	1,04	1,04	1,03	1,03	1,02
	$h = 8$	**1,00**	1,16	1,15	1,02	1,02	1,01	1,15	1,03	1,04	1,09	1,09	1,09	1,10
	$h = 12$	1,00	**0,92**	0,93	1,01	1,05	1,04	0,93	1,06	1,06	1,07	1,08	1,06	1,06
	$h = 24$	1,00	**0,89**	1,07	1,00	1,01	1,00	1,07	1,10	1,05	1,32	2,72	2,05	1,21
48	$h = 1$	**1,00**	1,00	1,00	1,00	1,00	1,30	1,00	1,19	1,19	1,19	1,19	1,19	1,19
	$h = 2$	**1,00**	1,16	1,16	1,15	1,15	1,20	1,16	1,09	1,09	1,11	1,10	1,10	1,11
	$h = 4$	**1,00**	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,02	1,02
	$h = 6$	**1,00**	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,03	1,03	1,02	1,02	1,02
	$h = 8$	**1,00**	1,17	1,16	1,02	1,02	1,13	1,16	1,03	1,03	1,08	1,09	1,09	1,10
	$h = 12$	1,00	**0,97**	0,99	1,08	1,12	1,24	**0,99**	1,12	1,12	1,12	1,13	1,13	1,13
	$h = 24$	1,00	**0,92**	1,10	1,04	1,05	1,17	1,10	1,05	1,09	1,08	1,10	1,10	1,14
60	$h = 24$	**1,00**	0,90	1,06	1,03	1,04	1,19	1,06	1,05	1,08	1,08	1,09	1,09	1,10

Table 2 - RMSFE of month on month inflation forecasts for the evaluation period between February of 2002 and September of 2011 produced using the simple AR as benchmark and the DAAR method with the breakeven inflation series with a maturity of 10 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

5.2 Quarterly inflation forecasts

Quarterly inflation was forecasted with $h = 1,2,3,4$ (a forecasting horizon of 1 period corresponds to the quarter during which the SPF is performed). As in the previous forecasting exercise, inflation forecasts using different breakeven inflation series yielded very similar results but the series with maturities of 5 and 7 years provided the best TIPS based forecasts of quarterly inflation. The relative RMSFE of the best forecasts of quarterly inflation are presented in the following table:

Forecast horizon	$h = 1$	$h = 2$	$h = 3$	$h = 4$
SPF	1,97	1,15	1,13	1,13
IAR (FL1)	**1,00**	**1,00**	1,03	1,04
IAR (FL2)	1,13	1,08	1,03	1,04
TIPS based (BEI05Y)	1,19	1,02	**1,00**	**1,00**
TIPS based (BEI07Y)	1,16	1,02	1,01	1,00

Table 3 - Relative RMSFE of quarterly inflation forecasts of SPF, IAR (FLX) (X is the number of fixed labs used) and TIPS based forecasts (BEIXXY) indicates the breakeven inflation series used) for the period of 2002Q1 until 2011Q3

The superior results of the IAR (FL) forecasts for $h = 1$ confirms the superior results of this autoregressive model for forecasting month-on-month inflation for short horizons expressed in tables 1 and 2. However, for horizons of 3 and 4 periods forecasts computed with inflation expectations extracted from the TIPS yield curve obtained the best results, proving their worthiness in forecasting month-on-month inflation with horizons between 6 and 11 periods. Both of these methods were able to beat the SPF forecasts for every horizon.

6 Conclusions

In this work project we have shown that using breakeven inflation series as additional regressors to simple autoregressive processes can improve forecasts of month-on-month inflation for some horizons. Besides that, for some horizons the forecasting errors of forecasts based on inflation expectations are very close to those of forecasts produced with the best method tested (iterated autoregression with fixed lag length).

Quarterly inflation forecasts derived from month-on-month inflation forecasts computed with a fixed lag iterated autoregressive method outperformed the SPF for horizons of 1 and 2 periods. On the other hand, forecasts backed by inflation expectations extracted from the TIPS yield curve outperformed SPF for horizons of 3 and 4 periods.

Estimating liquidity and inflation risk premiums would provide a more accurate measure of expected inflation than breakeven inflation series and using them as additional regressors should improve the forecasts performed in this work project. This would be of considerable importance in the case of month-on-month inflation forecasts as they are slightly outperformed by iterated autoregressive with fixed lag length.

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Appendix A – RMSFE of Quarterly Inflation Forecasts

Forecast Horizon	$h = 1$	$h = 2$	$h = 3$	$h = 4$
SPF	0,61	0,76	0,76	0,76
TIPS based (BEI05Y)	0,37	0,67	0,67	0,67
TIPS based (BEI06Y)	0,36	0,67	0,68	0,67
TIPS based (BEI07Y)	0,36	0,67	0,68	0,67
TIPS based (BEI08Y)	0,37	0,67	0,69	0,68
TIPS based (BEI09Y)	0,37	0,67	0,69	0,67
TIPS based (BEI10Y)	0,37	0,68	0,71	0,69
TIPS based (BEI11Y)	0,36	0,68	0,71	0,69
TIPS based (BEI12Y)	0,36	0,68	0,71	0,69
TIPS based (BEI13Y)	0,36	0,68	0,71	0,70
TIPS based (BEI14Y)	0,36	0,68	0,71	0,70
TIPS based (BEI15Y)	0,36	0,68	0,71	0,70
TIPS based (BEI16Y)	0,36	0,68	0,71	0,70
TIPS based (BEI17Y)	0,36	0,68	0,70	0,70
TIPS based (BEI18Y)	0,36	0,68	0,70	0,70
TIPS based (BEI19Y)	0,36	0,68	0,70	0,70
TIPS based (BEI20Y)	0,36	0,68	0,70	0,70
IAR FL1	0,31	0,66	0,69	0,70
IAR FL2	0,35	0,71	0,69	0,70
IAR FL4	0,35	0,75	0,69	0,70
IAR FL6	0,37	0,80	0,66	0,70
IAR FL8	0,37	0,83	0,73	0,68

Table 4 - RMSFE of quarterly inflation forecasts of SPF, IAR (FLX) (X is the number of fixed labs used) and TIPS based forecasts (BEIXXY indicates the breakeven inflation series used) for the period of 2002Q1 until 2011Q3

Appendix B – Relative RMSFE of Month-onMonth forecasts with autoregressive models and breakeven inflation series

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,18	1,18	1,18	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,09	1,13	1,13	1,14	1,13	1,13
	$h = 4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,03	1,03	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,03	1,03	1,03	1,05	1,05	1,05
	$h = 8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,04	1,04	1,15	1,15	1,16	1,18
	$h = 10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,06	1,06	1,07	1,06	1,07	1,11
	$h = 12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	0,98	1,11	1,81	4,75	2,28	1,84
48	$h = 1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,17	1,18	1,18	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,09	1,09	1,09	1,10	1,10	1,10
	$h = 4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,03	1,02	1,02	1,03	1,02	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,03	1,03	1,03	1,03	1,04
	$h = 8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,03	1,03	1,14	1,14	1,15	1,16
	$h = 10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,13	1,12	1,13	1,13	1,13	1,13
	$h = 12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,02	1,11	1,17	1,18	1,18	1,55
60	$h = 24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,01	1,09	1,16	1,16	1,17	1,17

Table 5 - Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with maturity of 7 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,18	1,18	1,18	1,18	1,18	1,18
	$h = 2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,09	1,12	1,12	1,12	1,13	1,13
	$h = 4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,03	1,03
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,03	1,04	1,04	1,05	1,05	1,05
	$h = 8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,03	1,04	1,13	1,14	1,15	1,16
	$h = 10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,06	1,06	1,07	1,06	1,07	1,11
	$h = 12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	0,98	1,10	2,40	2,75	1,99	1,85
48	$h = 1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,18	1,18	1,18	1,18	1,18	1,18
	$h = 2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,09	1,08	1,08	1,08	1,10	1,10
	$h = 4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,02	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,03	1,03	1,04	1,04	1,03
	$h = 8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,03	1,03	1,12	1,13	1,13	1,14
	$h = 10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,13	1,12	1,13	1,13	1,13	1,13
	$h = 12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,03	1,10	1,15	1,16	1,16	1,54
60	$h = 24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,02	1,08	1,14	1,14	1,15	1,15

Table 6 - Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with maturity of 6 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,18	1,18	1,18	1,18	1,18	1,18
	$h = 2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,08	1,12	1,13	1,13	1,13	1,14
	$h = 4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,03	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,03	1,04	1,04	1,04	1,04	1,04
	$h = 8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,03	1,04	1,12	1,12	1,13	1,14
	$h = 10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,06	1,06	1,06	1,06	1,06	1,10
	$h = 12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	0,98	1,08	4,74	1,67	1,81	1,77
48	$h = 1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,18	1,18	1,18	1,18	1,18	1,18
	$h = 2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,08	1,08	1,10	1,10	1,10	1,10
	$h = 4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,02	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,03	1,03	1,03	1,03	1,03
	$h = 8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,03	1,03	1,11	1,11	1,11	1,12
	$h = 10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,13	1,12	1,13	1,13	1,13	1,13
	$h = 12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,03	1,09	1,13	1,14	1,14	1,50
60	$h = 24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,02	1,08	1,12	1,13	1,13	1,14

Table 7 - Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with maturity of 7 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,18	1,18	1,18	1,18	1,19	1,18
	$h = 2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,09	1,12	1,14	1,14	1,14	1,14
	$h = 4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,04	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,03	1,04	1,04	1,04	1,04	1,03
	$h = 8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,03	1,04	1,10	1,11	1,11	1,12
	$h = 10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,06	1,06	1,07	1,06	1,06	1,11
	$h = 12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	0,99	1,07	6,52	1,42	1,68	2,17
48	$h = 1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,18	1,18	1,18	1,18	1,19	1,18
	$h = 2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,09	1,08	1,10	1,10	1,10	1,10
	$h = 4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,02	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,03	1,03	1,03	1,03	1,02
	$h = 8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,03	1,03	1,10	1,10	1,10	1,11
	$h = 10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,12	1,12	1,13	1,13	1,13	1,13
	$h = 12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,04	1,09	1,12	1,12	1,13	1,84
60	$h = 24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,03	1,08	1,11	1,11	1,12	1,12

Table 8 - Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with a maturity of 8 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,19	1,19	1,19	1,18	1,18	1,18
	$h = 2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,09	1,12	1,13	1,13	1,14	1,14
	$h = 4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,04	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,03	1,04	1,04	1,03	1,03	1,03
	$h = 8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,03	1,04	1,09	1,10	1,10	1,11
	$h = 10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,06	1,06	1,07	1,06	1,06	1,05
	$h = 12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	1,00	1,06	1,89	1,48	1,59	10,09
48	$h = 1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,19	1,19	1,20	1,18	1,18	1,19
	$h = 2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,09	1,08	1,10	1,10	1,10	1,10
	$h = 4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,02	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,03	1,03	1,02	1,02	1,02
	$h = 8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,03	1,03	1,09	1,09	1,09	1,10
	$h = 10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,12	1,12	1,13	1,13	1,13	1,13
	$h = 12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,04	1,09	1,10	1,10	1,11	1,13
60	$h = 24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,03	1,08	1,09	1,09	1,10	1,11

Table 9 - Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with maturity of 9 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,19	1,19	1,19	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,09	1,13	1,15	1,13	1,13	1,15
	$h = 4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,03	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,03	1,04	1,04	1,03	1,03	1,02
	$h = 8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,03	1,04	1,09	1,09	1,09	1,10
	$h = 10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,06	1,06	1,07	1,08	1,06	1,06
	$h = 12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	1,10	1,05	1,32	2,72	2,05	1,21
48	$h = 1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,19	1,19	1,19	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,09	1,09	1,11	1,10	1,10	1,11
	$h = 4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,02	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,03	1,03	1,02	1,02	1,02
	$h = 8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,03	1,03	1,08	1,09	1,09	1,10
	$h = 10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,12	1,12	1,12	1,13	1,13	1,13
	$h = 12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,05	1,09	1,08	1,10	1,10	1,14
60	$h = 24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,05	1,08	1,08	1,09	1,09	1,10

Table 10 - Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with maturity of 10 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,18	1,19	1,19	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,09	1,13	1,15	1,15	1,15	1,15
	$h = 4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,03	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,03	1,04	1,04	1,04	1,03	1,05
	$h = 8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,04	1,04	1,08	1,07	1,07	1,10
	$h = 10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,05	1,06	1,07	1,09	1,11	1,09
	$h = 12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	1,01	1,05	1,15	2,22	3,64	1,24
48	$h = 1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,19	1,19	1,19	1,19	1,20	1,19
	$h = 2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,09	1,09	1,11	1,11	1,11	1,11
	$h = 4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,02	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,03	1,04	1,04	1,02	1,05
	$h = 8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,03	1,04	1,07	1,07	1,07	1,10
	$h = 10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,12	1,12	1,12	1,13	1,13	1,13
	$h = 12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,06	1,08	1,08	1,09	1,09	1,15
60	$h = 24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,05	1,07	1,07	1,08	1,08	1,10

Table 11 - Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with maturity of 11 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,18	1,18	1,18	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,09	1,13	1,15	1,14	1,15	1,15
	$h = 4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,03	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,03	1,04	1,04	1,05	1,05	1,05
	$h = 8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,03	1,04	1,06	1,07	1,07	1,10
	$h = 10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,05	1,05	1,07	1,09	1,10	1,14
	$h = 12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	1,02	1,05	1,09	4,05	43,97	1,10
48	$h = 1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,19	1,19	1,19	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,09	1,09	1,11	1,11	1,11	1,11
	$h = 4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,02	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,04	1,04	1,05	1,05	1,05
	$h = 8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,03	1,03	1,06	1,06	1,06	1,09
	$h = 10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,12	1,12	1,12	1,13	1,13	1,13
	$h = 12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,06	1,08	1,08	1,08	1,08	1,10
60	$h = 24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,06	1,08	1,07	1,08	1,08	1,08

Table 12 - Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with maturity of 12 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,18	1,18	1,18	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,10	1,13	1,15	1,14	1,15	1,15
	$h = 4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,04	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,03	1,04	1,06	1,05	1,05	1,07
	$h = 8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,04	1,04	1,06	1,07	1,06	1,09
	$h = 10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,05	1,05	1,07	1,09	1,09	1,68
	$h = 12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	1,03	1,04	1,07	2,36	5,47	1,09
48	$h = 1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,18	1,19	1,19	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,10	1,09	1,11	1,11	1,11	1,11
	$h = 4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,02	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,04	1,06	1,05	1,05	1,07
	$h = 8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,03	1,03	1,06	1,06	1,06	1,08
	$h = 10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,12	1,12	1,12	1,13	1,13	1,13
	$h = 12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,07	1,08	1,09	1,08	1,07	1,10
60	$h = 24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,06	1,07	1,08	1,07	1,07	1,07

Table 13 - Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with maturity of 13 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h=1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,18	1,18	1,18	1,19	1,19	1,19
	$h=2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,10	1,13	1,15	1,14	1,15	1,15
	$h=4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,04	1,03
	$h=6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,04	1,04	1,06	1,07	1,06	1,07
	$h=8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,03	1,04	1,06	1,06	1,06	1,08
	$h=10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,05	1,05	1,07	1,09	1,09	2,20
	$h=12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	1,03	1,05	1,06	1,39	5,95	1,06
48	$h=1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,18	1,19	1,19	1,19	1,19	1,19
	$h=2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,10	1,09	1,11	1,11	1,11	1,11
	$h=4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,02	1,02
	$h=6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,03	1,06	1,07	1,06	1,07
	$h=8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,03	1,03	1,05	1,05	1,06	1,08
	$h=10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,12	1,12	1,12	1,13	1,13	1,13
	$h=12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,08	1,09	1,08	1,09	1,07	1,07
60	$h=24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,06	1,08	1,07	1,07	1,06	1,07

Table 14 - Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with maturity of 14 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,18	1,18	1,18	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,10	1,12	1,15	1,14	1,15	1,15
	$h = 4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,03	1,03
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,04	1,05	1,06	1,08	1,06	1,07
	$h = 8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,03	1,04	1,06	1,06	1,06	1,07
	$h = 10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,05	1,05	1,07	1,08	1,08	3,03
	$h = 12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	1,04	1,04	1,05	1,28	11,50	1,07
48	$h = 1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,19	1,19	1,19	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,10	1,09	1,11	1,11	1,11	1,11
	$h = 4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,02	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,05	1,06	1,08	1,06	1,07
	$h = 8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,03	1,03	1,05	1,05	1,05	1,06
	$h = 10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,12	1,12	1,12	1,12	1,13	1,12
	$h = 12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,08	1,08	1,09	1,10	1,07	1,08
60	$h = 24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,06	1,07	1,07	1,08	1,06	1,06

Table 15 - Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with maturity of 15 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,18	1,18	1,19	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,10	1,12	1,15	1,14	1,15	1,15
	$h = 4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,03	1,03
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,04	1,05	1,06	1,06	1,06	1,06
	$h = 8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,04	1,04	1,05	1,06	1,06	1,06
	$h = 10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,05	1,05	1,07	1,08	1,08	1,72
	$h = 12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	1,05	1,03	1,05	1,26	34,60	1,07
48	$h = 1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,19	1,19	1,19	1,19	1,20	1,19
	$h = 2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,10	1,09	1,11	1,11	1,11	1,11
	$h = 4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,03	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,05	1,06	1,06	1,06	1,06
	$h = 8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,03	1,03	1,05	1,05	1,05	1,05
	$h = 10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,12	1,12	1,12	1,12	1,13	1,12
	$h = 12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,09	1,07	1,09	1,10	1,07	1,07
60	$h = 24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,06	1,05	1,07	1,08	1,06	1,06

Table 16 - Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with maturity of 16 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,19	1,19	1,19	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,09	1,13	1,15	1,15	1,13	1,15
	$h = 4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,03	1,04
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,04	1,05	1,05	1,06	1,06	1,05
	$h = 8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,04	1,04	1,05	1,06	1,06	1,06
	$h = 10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,05	1,06	1,06	1,07	1,07	1,07
	$h = 12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	1,06	1,03	1,05	1,26	7,44	1,08
48	$h = 1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,19	1,19	1,19	1,19	1,19	1,20
	$h = 2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,09	1,09	1,11	1,11	1,09	1,11
	$h = 4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,02	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,05	1,05	1,06	1,05	1,05
	$h = 8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,02	1,03	1,04	1,05	1,05	1,05
	$h = 10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,12	1,12	1,12	1,12	1,12	1,12
	$h = 12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,10	1,09	1,09	1,11	1,07	1,08
60	$h = 24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,06	1,07	1,07	1,08	1,06	1,06

Table 17- Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with maturity of 17 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,19	1,19	1,19	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,09	1,13	1,15	1,13	1,13	1,15
	$h = 4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,03	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,03	1,04	1,04	1,03	1,03	1,02
	$h = 8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,03	1,04	1,09	1,09	1,09	1,10
	$h = 10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,06	1,06	1,07	1,08	1,06	1,06
	$h = 12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	1,07	1,06	1,05	1,26	4,93	1,09
48	$h = 1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,19	1,19	1,19	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,09	1,09	1,11	1,10	1,10	1,11
	$h = 4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,02	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,03	1,03	1,02	1,02	1,02
	$h = 8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,03	1,03	1,08	1,09	1,09	1,10
	$h = 10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,12	1,12	1,12	1,13	1,13	1,13
	$h = 12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,05	1,09	1,08	1,10	1,10	1,14
60	$h = 24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,05	1,08	1,08	1,09	1,09	1,10

Table 18 - Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with a maturity of 18 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,19	1,19	1,19	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,09	1,13	1,15	1,13	1,13	1,15
	$h = 4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,03	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,03	1,04	1,04	1,03	1,03	1,02
	$h = 8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,03	1,04	1,09	1,09	1,09	1,10
	$h = 10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,06	1,06	1,07	1,08	1,06	1,06
	$h = 12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	1,10	1,07	1,06	1,26	6,30	1,11
48	$h = 1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,19	1,19	1,19	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,09	1,09	1,11	1,10	1,10	1,11
	$h = 4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,02	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,03	1,03	1,02	1,02	1,02
	$h = 8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,03	1,03	1,08	1,09	1,09	1,10
	$h = 10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,12	1,12	1,12	1,13	1,13	1,13
	$h = 12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,05	1,09	1,08	1,10	1,10	1,14
60	$h = 24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,05	1,08	1,08	1,09	1,09	1,10

Table 19 - Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with maturity of 19 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)

Starting Observation (SO)	Horizon	IAR (FL)	DAR (FL)	DAR (AIC)	IAR (AIC)	IAR(BIC)	No-change	AR Benchmark	DAAR (ML2)	DAAR (ML4)	DAAR (ML6)	DAAR (ML8)	DAAR (ML10)	DAAR (ML12)
36	$h = 1$	1,00	1,00	1,00	1,00	1,01	1,16	1,00	1,19	1,19	1,19	1,19	1,19	1,19
	$h = 2$	1,00	1,16	1,16	1,14	1,14	1,07	1,16	1,09	1,12	1,13	1,13	1,13	1,14
	$h = 4$	1,00	1,11	1,15	1,05	1,05	1,01	1,15	1,02	1,02	1,02	1,02	1,03	1,04
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,02	1,05	1,04	1,04	1,05	1,05	1,04	1,03
	$h = 8$	1,00	1,16	1,15	1,02	1,02	1,01	1,15	1,04	1,04	1,03	1,03	1,03	1,06
	$h = 10$	1,00	0,92	0,93	1,01	1,05	1,04	0,93	1,05	1,06	1,06	1,06	1,07	1,05
	$h = 12$	1,00	0,89	1,07	1,00	1,01	1,00	1,07	1,10	1,07	1,06	1,26	6,30	1,11
48	$h = 1$	1,00	1,00	1,00	1,00	1,00	1,30	1,00	1,19	1,20	1,20	1,19	1,20	1,20
	$h = 2$	1,00	1,16	1,16	1,15	1,15	1,20	1,16	1,09	1,09	1,09	1,09	1,09	1,10
	$h = 4$	1,00	1,12	1,15	1,05	1,05	1,13	1,15	1,02	1,02	1,02	1,02	1,02	1,02
	$h = 6$	1,00	1,05	1,05	1,00	1,01	1,14	1,05	1,03	1,03	1,05	1,05	1,03	1,03
	$h = 8$	1,00	1,17	1,16	1,02	1,02	1,13	1,16	1,02	1,02	1,02	1,02	1,02	1,05
	$h = 10$	1,00	0,97	0,99	1,08	1,12	1,24	0,99	1,12	1,12	1,12	1,12	1,12	1,12
	$h = 12$	1,00	0,92	1,10	1,04	1,05	1,17	1,10	1,12	1,10	1,10	1,11	1,08	1,08
60	$h = 24$	1,00	0,90	1,06	1,03	1,04	1,19	1,06	1,08	1,07	1,08	1,08	1,07	1,06

Table 20 - Relative RMSFE of month-on-month inflation forecasts produced with benchmark autoregressive methods and direct augmented autoregressive forecasts with breakeven inflation with maturity of 20 years (MLX denotes the number of maximum lags parameter of AIC lag length selection in DAAR method)